Using metaphors as lenses to understand fifth-grade students' conceptualisation of fraction sub-constructs

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In this paper, we use video documentation to examine fifth-grade students' use of metaphors as they in small groups in a whole-class setting are engaged in a practical learning activity about fractions in the everyday-lives context of a vegetable garden. The aim of our study is to understand how the students collaboratively conceptualise different sub-constructs of fractions. In our analysis, we link metaphor analysis with fraction sub-constructs as described in the literature. From the verbal and non-verbal language as employed by the students, we deduced three conceptual metaphors, namely `fractions as containers', 'fractions as separating parts', and 'fractions as sizes', as indicators for students' understanding of the fraction constructs. Based on our findings, we conclude that using metaphors as lenses can help getting a better grasp of students' understanding of fraction constructs.

Keywords: Conceptual metaphor, fraction sub-constructs, fifth-grade students, practical activity.

Introduction

Understanding fractions is a key competence in primary and secondary mathematics education worldwide. There is consensus in the mathematics community this competence lies the groundwork for grasping other mathematical topics, for example algebra (Bailey et al., 2012), which again are fundamental for all STEM-related disciplines.

In Norway, a new curriculum reform in 2020 (Utdanningsdirektoratet, 2020) was introduced with the overall aim to facilitate intra-and interdisciplinary deep learning processes where students are to develop a collection of cognitive and practical competencies in order to be able to transfer knowledge between subjects and between their school and everyday lives (Pellegrino & Hilton, 2012). In line with this reform the fraction curriculum for the 5th grade emphasises, amongst other things, that students have to develop the competence to describe fractions as a part of a whole and a part of a quantity, and as a number on the number line along with the competence to represent fractions in different ways (Utdanningsdirektoratet, 2020). Furthermore, students are supposed to formulate and solve problems related to fractions from their own everyday lives.

Fractions are a complex mathematical concept which continuously challenges students in developing conceptual understanding and teachers to facilitate learning processes leading to this (Berggren, 2022). It is claimed that these challenges on the one hand mainly relate to the content which is taught, namely the part-whole concept, which has a tendency to gain most attention in education (Wilkins & Norton, 2018) as one out of five identified fraction sub-constructs (Kieren, 1980). On the other hand, it is reported that students have difficulties understanding the multifaceted nature of fractions, that is understanding and linking the sub-constructs to each other (Lamon, 2020). Pantziara and Philippou (2012, p. 63) describe the sub-constructs as follows:

> The fraction 3/4 can be conceived as a part of a whole (three out of four equal parts), as a quotient (three divided by four), an operator (three quarters of a quantity), a ratio (three parts to four parts), and finally as measure (as a point on a number line).

The goal of the present study is to identify which and how these sub-constructs are conceptualised by using students' employed metaphors as an analytical lens. For this, we used a video recording of a small group of students collaboratively working with a practical learning activity.

Conceptual framework and research questions

Metaphors, as we consider them are not merely a stylistic device, but a central experience-based conceptual process which allows us to understand one concept in terms of another (Lakoff & Núñez 2000; Vosniadou & Ioannides, 1998). As it is argued in the context of cognitive linguistics (Pantziara & Philippou, 2012), understanding mathematics can be considered an abstract area of thought that deeply relies on experiential grounding. For example, as explained by Johnson (2005, p. 26 ff.), it seems that we conceptualise arithmetic operations based on the conceptual metaphor "Arithmetic IS motion along a path". When, for example, saying one is "greater than" zero, we imagine zero as the origin point and one as a "point-location on that path which is further away from that origin than zero. Analogically, a simple fraction (1/*n*) can be understood as, "starting at 1, finding the distance such that by moving distance *d* toward the origin repeatedly *n* times, you will reach the origin" (Berggren, 2022, p. 4). Conceptual Metaphor Theory (CMT) as illustrated by these examples, proposes the idea to understand our cognitive processes as inseparably linked to the language we use and the experiences we make through our everyday lives. Combined with the idea that learning is a social process where existing conceptions are actively reconstructed (Vosniadou & Ioannides, 1998), CMT provides a powerful theory to reconstruct students' understanding of abstract concepts which often are perceived as difficult by students.

With regard to students' conceptualisation of fractions being a major area of mathematics education research (Charalambous & Pitta-Pantazi, 2007) and an expressed need for more research which can help teachers facilitate students' learning with metaphors in mathematics education in general (Soto-Andrade, 2020), our study aims to produce knowledge about students' conceptualising of fractions through metaphors. Earlier research has shown that CMT can be a conducive framework to study fractions, for example, as they are described in school textbooks (Berggren, 2022), as they are used by students when attempting to conceptualise improper fractions in a written task (Ahn, 2022) and when they are—intuitively—employed by students in whole classroom lessons (Wood, 2010). Interestingly, Wood (2010) identified that by using different metaphors, such as *Fractions as pieces* (as pieces of a whole) and *Fractions as a container* (as distinct containers to be filled with numbers) different students' understanding of fractions differs even when working on the same task.

According to the considerations above the aim of this study is to examine fifth-grade students' use of metaphors to elucidate their conceptual understanding of fractions as they collaboratively are being engaged with a practical learning activity. The idea is to provide a detailed look into a students' meaning-making processes as they, in a group of three, jointly interact with a practical activity (they sit on a green carpet representing a vegetable bed on which they are supposed to cultivate different vegetables). The research questions that guided our study were the following:

- What kind of conceptual metaphors do fifth-grade students use when collaboratively engaging in a practical learning activity?
- What do the used conceptual metaphors reveal about the students' understanding of the fraction sub-constructs?

Methods

Design and students involved

For this study we used part of the video-recordings that were made in the Newton Rooms project (https://www.nord.no/en/about/faculties-and-centres/faculty-of-education-and-arts/research/research-groups/learning-in-interaction), which aim is to investigate how students can develop deep learning in STEM subjects when they are working in groups on activity-rich and inquiry-based learning activities. Using the high-quality video-recordings of the Newton Rooms project offered us the opportunity to clearly identify the speakers in the group talk and the meaning-making processes, and it also informed us about the non-verbal language and the students' positioning while working with the activity (Rusk et al., 2015). The analysed data reported in this paper involve the communication of a group of three fifth-grade students, who all provided a written consent to participate in the study. In the paper they are mentioned Anna, Bianca and Carina. The students are from one school nearby the Newton room where the out-of-school learning activities take place.

The "Vegetable-bed learning activity"

Out of the learning activities designed by the two teachers of the Newton room, we chose for our study the so-called "vegetable-bed learning activity" (Table 1).

Table 1: Vegetable-bed learning activity

Instruction	Material	Duration
• The green carpet is the vegetable bed. Additionally, there is a basket of material free to dispose. Use it if you need it. • Plant potatoes in half of the vegetable bed. • Cultivate carrots in two thirds of what is left after the potatoes. • Plant onions in halve of what is left. • Plant cabbage in the rest of the bed. What is the size of the remaining part where the cabbage is supposed to be?	• Written note with instructions. • Green carpet sized approximately 1,5 x 1m as vegetable bed model. • Laminated pictures of the four vegetables. • Tape.	ca. 40 minutes

The reason why we chose this activity was due to our research question. Compared to the other learning activities in this module, we observed that this activity elicited a lot of spontaneous context-based talks among the students. This gave us the opportunity to detect traces of metaphors used by the students, in their verbal as well as non-verbal language related to fractions: The activity was based on learning material using everyday objects, such as a green carpet illustrating a vegetable bed (see a more detailed description below), thus inviting the students to bodily interactions while working with it. Furthermore, the instruction given (Table 1) was very open in terms of providing the students with the freedom to come with their own solutions and—equally important—to use their own wordings.

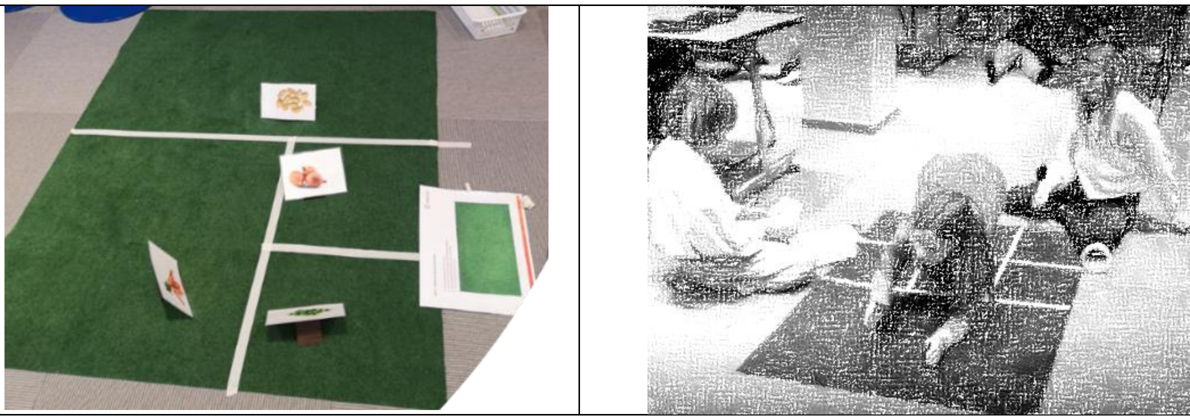

Figure 1: (left) The resulting vegetable bed; (right) Screenshot showing the girls using their hands

The students were sitting on the ground around a carpet that is supposed to be the vegetable bed. They worked out where to cultivate four different vegetables (laminated pictures of them) according to the given instruction in different parts of the vegetable bed and, subsequently, to identify the size of the part which was left for the last vegetable (Figure 1 on the left). During the learning activity, the girls used, apart from spoken words, parts of their bodies, such as their fingers and hands, to solve the task (Figure 1 on the right).

Analysis

The data analysis started with familiarisation with the video recording of the students' work. Then followed making a verbatim transcription of them. Since the aim was to identify metaphors in students' meaning-making processes, it was crucial to remain as close as possible to the original data (Kvale & Brinkmann, 2009). Therefore, no cleaning of grammar or dialect was foretaken, but students' exact phrasing was kept, as were breaks in the conversation and non-verbal gestures (such as using the hands to express a conception) and other non-verbal language (such as laughter).

The identification of metaphors was based on students' verbal utterances, non-verbal actions and gestures. This process was guided by a systematic metaphor analysis according to Schmitt (2017) and Lakoff & Núñez (2000). The analysis process was shaped by both deductive and inductive categorisation seeking for semantical structures, such as prepositions, conjunctions (proposing the presence of analogies, verbs etc. As an example, prepositions such as *into* or *in* indicate the presence of the *container* metaphor (Johnson, 2005). For the deductive approach, we used the sub-constructs of fractions as described in the previous sections of this article (Pantziara & Philippou, 2012; Kieren, 1980) along with conceptual metaphors as proposed by Lakoff and Núñez (2000) and Berggren (2022) as overall lenses to categorise students' utterances. The inductive approach was based on first identifying and then grouping metaphors directly out of the data, and then, in a process of excessive discussion, labelling these with a metaphor we found appropriate. In this process, we frequently consulted the online lexicon (Lexico.com, 2019) in order to better understand the (different), usually commonly shared meaning, of terms.

The analysis was mainly carried out by the first author. The subsequent articulation and interpretation of findings was then excessively discussed among the authors as was the selection of representative quotes. In the process of this, all data cited in this article, was translated from Norwegian to English with the utmost care to maintain the meaning of students' utterances.

Findings

Our analysis revealed that the students during their engagement with the learning activity appear to conceptualise fractions according to three out of five different subconstructs, namely the subconstructs part-whole, measure and quotient. The identification of different conceptual metaphors suggests that all of these subconstructs are rooted in different experiential source domains, thereby metaphors linked to the part-whole subconstruct being the most prevalent ones. Linked to the three subconstructs, we inductively and deductively (Berggren, 2022; Lakoff & Núñez, 2000) identified the following three conceptual metaphors: *fractions as containers*, *fractions as separated parts*, and *fractions as sizes*. In Table 2 we summarise the linkage between all identified sub-constructs and metaphors.

Table 2: Examples of identified metaphors, underlying experiential grounding, fraction sub-constructs

Examples	Conceptual Metaphor	Explanation of the experiential grounding of the metaphor	Understanding of fraction sub-construct
Anna: "**In** the rest of the land, it shall be cabbage." Bianca: "It is probably **here**."	Fraction as containers	Through using the prepositions **in, here** and **there,** the girls intuitively activate a sense of **containment**, hence conceiving of fractions as distinct unit parts filling the inside of the same area.	Part-whole
Carina: "We must **divide** it **in** three **equal parts**."	Fractions as separated parts	By using the words **divide** and **equal parts**, the girls intuitively invoke a sense of **partitioning**, conceptualising fractions as distinct units into which the whole can be partitioned.	Quotient
Anna: "We need to **find half**." Bianca: "Okay, half. Do we have something to **measure**? We can **cut**."	Fractions as sizes	By saying **finding half**, and **measuring something,** the girls intuitively invoke a sense of **construction**, conceiving **half** as a measurable size.	Measure

Fractions as containers and fractions as separated parts

Excerpt 1 illustrates how the three girls used the *fraction as containers* and *fraction as separated parts* metaphors to conceptualise the part-whole and quotient sub-constructs.

Excerpt 1

1	Anna:	You shall cultivate onions **in** half of what is left. Shall we **divide it** (the part of the bed which is left after partitioning for the carrots) **in**[1] the middle?
2	Bianca:	It is probably **here**.
3	Anna:	And **there** the onions shall be. And **in** the **rest of the land** then shall be cabbage.

In Line 1 and 3 Anna employs the preposition **in** while Bianca and Anna in Line 2 and 3 respectively use the adverb **here** and **there.** Furthermore, Anna uses the verb **divide** (Line 1), respectively the noun **rest** (line 3). We deduce from the language used by the girls that, instead of explicitly using the word container or part, the girls intuitively draw on experiences with containment and partitioning.

[1] Text highlighted in bold refers to the identified metaphors. Normal text in parenthesis highlights gestures. Italic text in parenthesis explains the context of the talk. Passages which are omitted are marked with three dots in a parenthesis (…).

In this way, they seem to project their familiarity with containing objects as distinct entities with an inside-outside orientation (such as their own body) and their experiences with the partitioning and subsequent separation of objects to structure their understanding of fractions. Consequently, the girls seem to conceptualise fractions as distinctively shaped areas (the different vegetable beds) which are separated from each other through a boundary (here the tape). Together, these areas fill the inside of the same area (the vegetable bed). Further, by drawing on their experiences with the partitioning and fitting together parts of everyday objects, such as when a cake or a puzzle is divided into smaller pieces, it seems that the girls conceive of fractions as physical objects being constituted of smaller parts (the vegetables) Together, these parts make up, but cannot exhaust the whole.

Fractions as sizes and fractions as containers

The excerpts below stem from the talk as the girls try to solve where to place the potatoes (the picture of these) according to the task to cultivate them on half the land The excerpt illustrates how the girls (Bianca) by using the metaphors *fractions as sizes* and *fractions as containers* attempt to understand the measure- and part-whole sub-constructs.

Excerpt 2

4	Anna:	potatoes **shall be on half the land**. Then we need **to find half**.
5	Bianca:	Okay, half, yes. Do we have **something to measure**?
6	Carina:	We can **cut.** (…after using the tape to separate the bed)
7	Bianca:	Then the **middle** is here, right?
8	Anna:	And then we take this (places the picture with the potatoes on one of the two areas they separated with the tape), right? The potatoes shall be **on** half of the land.
9	Bianca:	But we can have potatoes (*the picture*) **on** here (she points directly to the tape). Because they need to be **on** half the land.
10	Anna:	Yes, but this **whole** part is (hovers with one hand over one of the separated areas while going back and forth with her hand).
11	Bianca:	But this (!) is half (points to where the tape separates the bed).
12	Anna:	The potatoes should be **here** *(in the separated area and not the tape).*

In Line 3 Anna argues according to the given instruction that potatoes shall be **on** half the land and that this half needs to be **found**. Carina and Bianca suggest subsequently to **use something to measure** (Line 5) or to **cut** the vegetable bed. The prepositions, verbs and gestures (Line 9, Bianca pointing at the tape) used by the girls indicate that they implicitly draw on such experiences as identifying the sizes of unit parts in order to, for example, fitting and splitting parts to build a Lego construction. Based on these experiences, Bianca seems to conceptualise "half" not as one out of two equally sized objects (the other half), but rather as the point where an object splits in two (the middle) (Line 9). In contrast to Bianca, Anna appears to combine the metaphors *fractions as sizes* with the metaphor *fractions as containers*, therefore understanding **half** as both a distinct area within the whole vegetable bed as well as a unit part to be measured. Consequently, Bianca and Anna disagree with regard to where the potatoes should be planted (Line 11 and 12).

Discussion

In line with existing literature (Ahn, 2022; Wood, 2010) our findings reveal that students use conceptual metaphors to conceive of sub-constructs of fractions. We found that, when working with the vegetable-bed-activity, the students employed the following metaphors: *fractions as containers*, *fractions as separated parts*, and *fractions as sizes*. This gives support to the idea that they conceptualise three out of five subconstructs mentioned in literature (e.g., Kieren, 1980), namely the part-whole, the measure and the quotient sub-constructs. Apparently, by means of using these metaphors, the students made implicit comparisons to their everyday experiences with physical objects, namely the containment (*in, here, there*), the partitioning (*divide in*to) and the construction of objects (*cut, find*) to understand fractions as the relation between distinct, physical unit parts. In corroboration of Wood's (2010) findings, do our results furthermore indicate that students combine different experiential sources to make sense of the fraction sub-constructs, and that the usage of different source domains can be a source for misunderstandings between the students (for example, a different conceptualisation of "half"). Our results indicate that the container metaphor seems to be more prevalent than other metaphors when students conceptualise fractions. This is not surprising given the variety of experiences we have with physical containers, such as our own body, as spatial objects we use to put in and out smaller objects (Wood, 2010). Since the container metaphor was linked to the part-whole-, and quotient-sub-construct, it can seem that these sub-constructs are easier to understand than the ratio-and quotient-sub-constructs; we could not identify metaphors connected to these, proposing that students have insufficient experiences with these (Berggren, 2022).

With regard to teaching fractions with metaphors, our findings give reason to propose that firstly, teachers should become aware of their students' used metaphors, and thus provide opportunities for students to express these. Our findings indicate in line with other researchers' findings (Crowder, 1996) that practical learning activities inviting for the usage of different modalities of language, such as verbal and non-verbal ones (e.g., gestures), but also their own "everyday language" might help students to better conceptualise complex concepts through metaphors. This can, for example, be by means of contextualising fractions with everyday phenomena, such as the vegetable garden used here. Limitations of our study concern the rather little amount of data used, making the knowledge produced in this paper not generalisable per se. However, by means of providing transparency regarding our description of methods, analytical processes and findings, we think that our piece of research can provide sufficient strength to inform fraction educators who are interested in better understanding the use of metaphors in fraction education.

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